Shipment Consolidation

Ellissa Verseput [*]

Abstract

In this paper, methods to realize cost savings for Airbus Helicopters France are considered. This company makes a lot of shipments on different origin-destination pairs. By consolidating different shipments, bigger shipments can be sent to which lower cost/kg apply. In order to schedule such consolidations when orders come in at short notice, different policies are considered and a mixed integer linear program is designed. These methods are implemented and tested on different test instances. Their performances on cost savings and running times are compared and this shows that the policies would be more practical for day-to-day use, but that the mixed integer linear program generates more cost savings when Airbus Helicopters France strictly has to meet its customer's preferences.

1 Introduction

Airbus Helicopters France (AH-F) has to deal with a lot of shipments to and from their customers. At the moment the planning of these shipments seems far from optimal and the shipments are not kept track of and planned in an organized way. AH-F has hired the DHL LLP team, a supply chain consulting team within DHL, to bring structure to the different shipments and make optimization recommendations for their shipments planning in the future.

AH-F makes use of different carriers to do their shipments, among them DHL itself. AH-F operates from four warehouses, in Marseille, Paris and Hong Kong, from and to which

[*] Ellissa Verseput received a bachelor degree in Econometrics & Operations Research at Maastricht University in 2015, where she currently takes the Master in the same field.
Contact: e.verseput@student.maastrichtuniversity.nl

export and import takes place. AH-F sends shipments both by road and by air. For each shipment the client currently determines the service type. The choice is between a quick and customer-friendly, the so called "aircraft on ground" (AOG), service and a standard routine service. The service type chosen determines the time the shipment takes to arrive on its destination (the lead time).

DHL LLP retrieved data about shipments that have been made during 2014. As often in practice, the data is very incomplete, which makes analysis more complicated. Nevertheless each shipment should in theory have a couple of properties. A shipment in the past data has an origin airport and a destination airport. Moreover, for each shipment the carrier that executed the shipment is known, as well as the shipment type (road or air), the service type (AOG or routine), weight of the parcel, pick up date, lead time and total paid costs.

The information given by the past data needs to be translated to a planning system that can be used in the future. So, knowing or expecting that certain shipments need to be made in the future, a framework needs to be created that can schedule these shipments in a better way than is done at the moment. Part of AH-F's business exists of orders that arrive at short notice before their deadlines. This makes it interesting to look into models and policies that work with orders that arrive at short notice. Such models and policies can potentially be tailored to the AH-F case in order to achieve cost savings. This is the problem which this paper will deal with.

The method for achieving cost savings in this paper is shipment consolidation. This means that individual shipments of the same origin-destination pair will be combined in order to achieve cost savings, as the combined total weight will fall in a lower cost/kg rate. Routing will be treated as given, assuming that only specific airport-to-airport origin-destination pairs are considered for consolidation. Hence, consolidation will be optimized per unique origin-destination pair. Also it will be assumed that the cheapest carrier and transport mode can be chosen in the future, so that one cost rate table always applies to a certain origin-destination pair. Strictly speaking, inventory costs also should be taken into account when consolidation is done, because orders will need to be stored for a longer period of time before they are sent together with other orders. However, AH-F did not provide any information about their inventory structure, so it is not considered for this paper.

In the following, first, an overview of past work on optimization models and policies of freight transportation with a focus on consolidation will be presented in Section 2. Then in Section 3, the consolidation problem that will be worked with in this paper is formulated, after which different consolidation policies for this problem will be introduced in Section 4. A mixed integer linear program formulation of the consolidation problem will be described in Section 5. Test instances will be created in Section 6 and their results will be presented in Section 7.

2 Literature review

Freight transportation optimization is an area of operations research and supply chain management that has been researched a lot in the past decades. Many companies nowadays have a business aspect that involves freight transportation. High efficiency and great service quality are the standard. So, improving freight transportation and finding new ways of cost savings are booming business. Crainic and Laporte (1997) give a very broad overview of all the different levels at which freight transportation can be optimized. Strategic planning, the long term planning horizon, involves issues like network design and the locating of facilities. In tactical planning, the medium term planning horizon, service network design and vehicle routing problems are at hand. For the short term operational planning, scheduling and the allocation of resources needs to be decided on. In this paper, optimizing the tactical and operational planning are considered, as consolidation policies are determined on the tactical level and the actual scheduling of shipments is done at the operational level.

Higginson and Bookbinder (1994) describe and analyze three consolidation policies that are used for combining shipments. The time policy consolidates until the first order that came in has reached a certain age. The quantity policy combines shipments until a certain quantity is accumulated. A time and quantity policy does the previous at the same time; whichever constraint (time or quantity) is binding first, determines when a consolidation is released. By simulation and trying different parameters, they draw conclusions in which case which policy performs best. Bookbinder and Higginson (2002) compare consolidation of stochastically incoming orders having a Poisson distribution with a general stochastic clearing system in order to obtain the maximum holding time and maximum quantity.

Çetinkaya and Bookbinder (2003) analytically derive optimal parameters for the time and quantity policy, again using that orders come in with a Poisson arrival process. Mutlu et al. (2010) also analytically derive parameters for a hybrid time and quantity policy. Both papers rely heavenly on stochastic properties given by the Poisson distribution.

Instead of applying the standard consolidation policies, Tyan et al. (2003) formulate an integer optimization program for different service policies, applied to a real life case. They solve the integer program by using integer optimization software (Lingo), which is possible because the instances are not very big. Attanasio et al. (2007) do a case study as well, where besides consolidation constraints, also bin packing constraints are at hand. They solve the problem by first solving an (infeasible) integer linear program and then iteratively making the simplified solution feasible. Song et al. (2008) also formulate an optimization problem for a specific consolidation problem and recognize that it is NP hard. They design a heuristic to solve the optimization problem and compare it to solution retrieved (very slowly) by CPLEX. Qin et al. (2014) do a case study on consolidation by designing a heuristic for a variant that takes different containers and different routes in account besides consolidation.

3 Problem formulation

Following the literature, consolidation policies can offer AH-F a practical and easy to implement framework for the future. Solving their consolidation problem with an exact approach, like a mixed integer linear program (MILP) is also an option, although in practice it will probably be harder to implement for day-to-day use. This paper will both look into consolidation policies and an MILP solution and compare their results. The policies and the MILP will be applied to the following problem description:

- We consider one origin-destination pair.
- At the beginning of every day, all new orders of that day come in together.
- Those orders always need a fixed number of days before they are ready for shipping.
- From the moment that orders are ready for shipping, they have (heterogeneous) deadlines for arriving on destination that have to be met.
- Every order has a weight.
- The customer has chosen a service type (AOG or routine), but AH-F could decide to neglect this chosen service type, and see which cost savings this could bring, especially to have some more freedom to build consolidations for the policies.
- The AOG and routine service both have a fixed lead time.
- The orders need to be sent as cheap as possible, but without violating their deadlines (and service type in case we consider this as something that cannot be neglected).

The transportation costs of the AOG service are higher than the routine service. The transportation cost function, a function that only has the total weight of the shipment as an input, is non-decreasing and piecewise linear, such that the cost/kg rates are non-increasing. There will be no limit opposed on the total weight of a shipment. For visualization, an abstract transportation cost function $f(w)$ description and a sketch look as follows:

$$f(w) = \begin{cases} MAX(P_0; w * p_0) & \text{if } q_0 < w \leq q_1 \\ MAX(P_1; w * p_1) & \text{if } q_1 < w \leq q_2 \\ ... & \\ MAX(P_n; w * p_n) & \text{if } q_n < w \end{cases} \tag{3.1}$$

$$with\, q_0 = 0,\ p_0 > p_1 > ... > p_n,\ P_0 < P_1 < ... < P_n,\ q_{j+1} * p_j = P_{j+1}$$

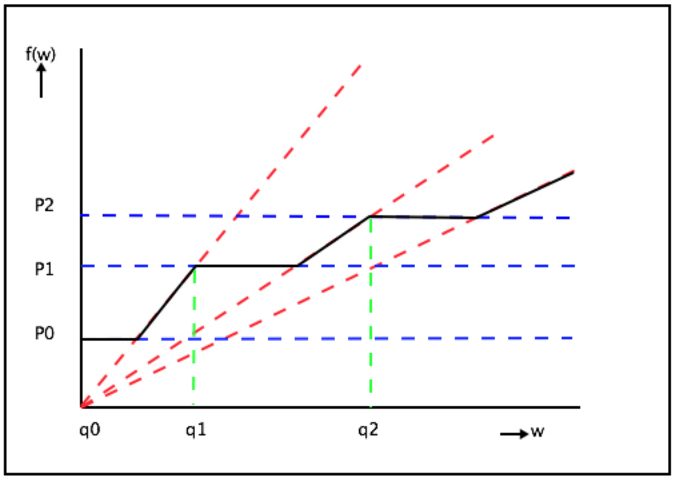

Figure 3.1: Sketch of $f(w)$

4 Consolidation policies

As described in section 2, Higginson and Bookbinder (1994) already pointed out different policies that can be used to consolidate shipments. These policies can potentially provide more structure and efficiency for AH-F. Take in mind that these policies should offer an easy to implement schedule advice for day-to-day use. So the policies described certainly do not lead to the most optimal schedule possible, but are just structural guidelines in order to achieve a decent consolidation scheme. Also notice that the fixed days between the day a shipment becomes known and is ready for shipping does not influence the policies described below.

4.1 Time policy

In the time policy described by Higginson and Bookbinder (1994), consolidation is done on the basis of the oldest order approach; As soon as the oldest order that is waiting to be shipped has reached a certain predetermined age T (in days), all the orders that are currently available to be shipped are consolidated and sent. However, the models in which a fixed T is used do not deal with heterogeneous deadlines. Hence, the time policy can not immediately be applied as such to AH-F, which also means that the analytical results of Çetinkaya and Bookbinder (2003) do not apply for determining the optimal T for a consolidation. Instead, tailor made time policies will be tested on the test instances. Given the lead times of the routine service type and the AOG service type, the deadline of an order that becomes binding first can be used to determine the time when a consolidation is shipped. Three policies are formulated:

AOG time policy In this policy, AH-F will choose the service type used and the AOG lead time is used to determine when a deadline becomes binding. So whenever the AOG lead time until the deadline of a certain order becomes the exact time left till the deadline, all orders that are currently waiting to be shipped are consolidated and sent. Under this policy, there is often more time to wait before shipping, as the AOG service is relatively faster. However, the AOG service is more expensive than the routine service. Nevertheless, as larger consolidations could be build up when shipping is delayed, the total weight will fall into a lower cost/kg rate, which could be cheaper than shipping smaller consolidations with the routine service.

Routine time policy In this policy the routine lead time determines when a deadline of a certain order becomes binding. This policy can profit from the lower cost rates of the routine service, while there is in general less time to build up a consolidation before a deadline becomes binding. If an order comes in which lead time is so short that it will not meet the deadline when it is shipped by the routine service, it will have to be shipped by the more expensive AOG service. As soon as this AOG lead time becomes binding, a couple of different sub-policies can be implemented:

- Routine time policy A: the urgent order that needs to be shipped by AOG is shipped separately. So all orders that are still feasibly shipped with the routine service are shipped in consolidation with the routine service. Although the AOG shipment will be relatively expensive, the routine consolidation is not touched and can hopefully still be sent with a lower cost/kg rate because of large weight and the lower routine service rates.

- Routine time policy B: all orders that are currently waiting are being shipped in consolidation with the urgent shipment with the AOG service. In this case, there is at least the potential for the AOG consolidation to fall into a lower cost/kg rate.

Customer chooses time policy Because the assumption that AH-F can neglect the service type chosen by the customer is quite strong, this policy will meet the customer's preferences (as the AOG service is not only faster, but it also comes with extra other services). So AOG and routine orders will have to be consolidated separately. Again, as soon as an AOG deadline will become binding, an AOG consolidation is sent with the current AOG orders and when a routine deadline will become binding a routine consolidation is sent.

Immediate shipping For comparing the above explained policies, the standard policy in which the incoming orders are just shipped right away with the preferred service type is implemented as well. In this policy, all orders that have come in on a certain day with preselected AOG service are immediately sent together and the same goes for the routine service. Notice that orders that have come in on the same day are still sent in consolidation, as the orders arrived together as well.

4.2 Quantity policy

The quantity policy described by Higginson and Bookbinder (1994), that sends out an consolidation whenever a certain target weight W has been accumulated, is not applicable to AH-F, as it cannot guarantee that the deadlines are met. Hence, implementing some kind of quantity policy does not make any sense.

5 Mixed integer linear program

The policies described in the foregoing section can provide cost savings already, but are not very sophisticated. They are just logical rules of thumb. Especially when there is no flexibility in neglecting the chosen service type, the policies do not have a lot of freedom to make consolidations. So in that case, something more sophisticated might be needed to provide substantial cost savings. The following MILP solution of the consolidation problem will take more computation time, but provides the optimal consolidation schedule seen from today's perspective. See Appendix A for a list of the used parameters and variables.

Considering a discrete rolling time horizon, the model will be optimized every day from the perspective of the current day $\bar{t}$, when new orders have come in. Remember it is assumed that at the beginning of each day t, all new orders that come in that day are known. At that moment, the scheduling and the consolidation of the current known orders is optimized and the orders that need to be sent today according to the current solution are then sent.

Let $I_{\bar{t}}$ be the set of orders that are known on the current day $\bar{t}$. Each $i \in I_{\bar{t}}$ has a couple of attributes. The arrival day: a_i, the number of days until the deadline after the order is ready for shipping: d_i, weight of the order: w_i and binary parameters for the service type selected by the customer: $k_{i,s}$, $s \in \{\text{AOG, routine}\}$. The lead times for the two service types, AOG and routine, are indicated with the parameters l_s, $s \in \{\text{AOG, routine}\}$. The fixed time between the arrival of an order and the day that the order is ready for shipping is the parameter r.

Every day, we need to plan from today $\bar{t}$ until day T, where $T = max_i\{a_i + r + d_i\}$. In order to assign each order $i \in I_t$ to a consolidation, binary decision variables $x_{i,t,s}$ are introduced, which equal 1 when order i is sent on day $t = \bar{t}, ..., T$ with service type $s \in$

{AOG, routine}. Because of the non-increasing cost/kg structure of the transportation cost described in equation (3.1), all orders that are sent on a certain day with the same service type, are best sent in one consolidation, as this will always be cheaper per kg than sending it with separate orders. That is why the decision variables $x_{i,t,s}$ are sufficient to determine the consolidations that are sent today and in the upcoming days.

The transportation cost function f of a consolidation of orders can be calculated using the total accumulated weight that is sent by it. So f's input is $\sum_{i \in I_{\bar{t}}}(x_{i,t,s} * w_i)$.

In principal the following integer program needs to be solved on current day $\bar{t}$:

$$\begin{aligned}
\text{Min} \quad & \sum_{s} \sum_{t=\bar{t}}^{T} f(\sum_{i \in I_{\bar{t}}}(x_{i,t,s} * w_i)) && (5.1)\\
\text{s.t.} \quad & \sum_{s} \sum_{t=\bar{t}}^{T} x_{i,t,s} = 1, \ \forall i \in I_{\bar{t}} && (5.2)\\
& \sum_{s} \sum_{t=\bar{t}}^{T} (t * x_{i,t,s}) \geq a_i + r, \ \forall i \in I_{\bar{t}} && (5.3)\\
& \sum_{s} \sum_{t=\bar{t}}^{T} ((t + l_s) * x_{i,t,s}) \leq a_i + r + d_i, \ \forall i \in I_{\bar{t}} && (5.4)\\
& \sum_{t=\bar{t}}^{T} x_{i,t,s} = k_{i,s} \ \forall i \in I_{\bar{t}}, \forall s && (5.5)\\
& x_{i,t,s} \in \{0, 1\} && (5.6)
\end{aligned}$$

(5.2) ensures that each order is assigned to only one consolidation. (5.3) makes sure that the orders are not sent before they are available. (5.4) takes care that the orders arrive before their deadlines. (5.5) ensures that the orders are sent by the correct service type.

However, the objective function (5.1) is not linear, so the integer program can not be solved like this. We need to add extra binary variables and modify the objective function in order to make this an integer linear program. Namely, problems with piece-wise linear objective functions like this can be formulated as mixed integer linear programs.

Before we do that, we first slightly change the presentation of the transportation cost function f. Remember in equation (3.1) we had:

$$f(w) = \begin{cases} MAX(P_0; w*p_0) & \text{if } q_0 < w \leq q_1 \\ MAX(P_1; w*p_1) & \text{if } q_1 < w \leq q_2 \\ ... & \\ MAX(P_n; w*p_n) & \text{if } q_n < w \end{cases}$$

$$with\, q_0 = 0,\ p_0 > p_1 > ... > p_n,\ P_0 < P_1 < ... < P_n,\ q_{j+1} * p_j = P_{j+1}$$

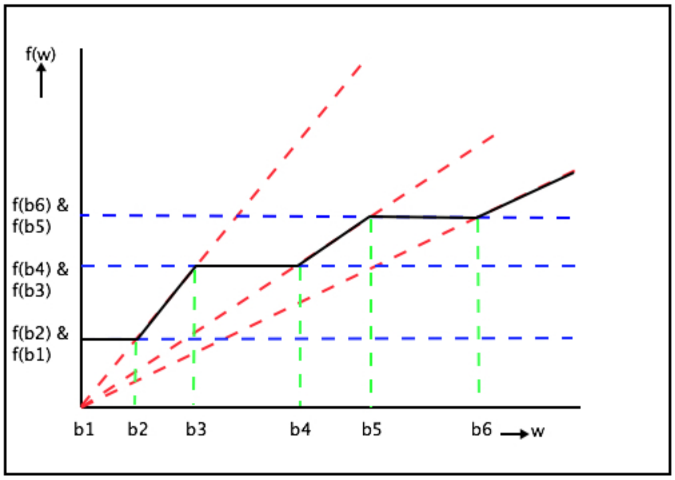

Figure 5.1: Sketch of new presentation of $f(w)$

Now consider the m fixed breakpoints of this function $b_1, ..., b_m$ and their fixed function value $f(b_1), ..., f(b_m)$, where $b_m = B$, with B chosen sufficiently large, such that no total weight of any consolidation will exceed B (see figure 5.1). Introducing additional real-valued variables $y_j \in [0, 1], j = 1, ..., m$, we can rewrite $f(w)$ as follows:

$$f(w) = \sum_{j=1}^{m} y_j * f(b_j) \tag{5.7}$$

$$\sum_{j=1}^{m} y_j * b_j = w \tag{5.8}$$

$$\sum_{j=1}^{m} y_j = 1 \tag{5.9}$$

(5.8) writes the original weight as a linear combination of the breakpoint weights and this makes sure that the cost for the original weight is calculated as a linear combination of the cost of the breakpoint weights in (5.9). (5.10) ensures that the y_js really make a linear combination by making them add up to 1. Now only one more thing is needed, namely that only two consecutive y_js are larger than 0. Otherwise, the linear combination will not represent a point on the original cost function line. In order to do this, we need the y_js to be so called "Special Ordered Set of type 2" variables (SOS2 variables), which simply means that only two consecutive y_js are larger than 0 in an ordered set of y_js. The additional mathematical constraints and binary variables z_js that ensure this are:

$$\sum_{j=1}^{m-1} z_j = 1 \tag{5.10}$$

$$y_j \leq z_{j-1} + z_j,\ j = 1, ..., m \tag{5.11}$$

$$z_j \in \{0, 1\}\ , z_0 = z_m = 0, y_j \in [0, 1] \tag{5.12}$$

(5.10) imposes that only one z_j can be equal to 1, say for $j = \bar{j}$. Hence (5.11) makes sure that only $y_{\bar{j}}$ and $y_{\bar{j}+1}$ can be larger than 0. These techniques have been retrieved from ideas explained by Bisschop (2006) and DAmbrosio (2010). Having introduced this new presentation of $f(w)$, we are ready to formulate the MILP again:

$$\text{Min} \quad \sum_{s} \sum_{t=\bar{t}}^{T} \sum_{j=1}^{m} (y_{j,t,s} * f(b_j)) \tag{5.13}$$

$$\text{s.t.} \quad \sum_{j=1}^{m} (y_{j,t,s} * b_j) = \sum_{i \in I_{\bar{t}}} (x_{i,t,s} * w_i), \ \forall s, t = \bar{t}, ..., T \tag{5.14}$$

$$\sum_{j=1}^{m} y_{j,t,s} = 1, \ \forall s, t = \bar{t}, ..., T \tag{5.15}$$

$$\sum_{j=1}^{m-1} z_{j,t,s} = 1, \ \forall s, t = \bar{t}, ..., T \tag{5.16}$$

$$y_{j,t,s} \leq z_{j-1,t,s} + z_{j,t,s}, \forall s, t = \bar{t}, ..., T, j = 1, ..., m \tag{5.17}$$

$$\sum_{s} \sum_{t=\bar{t}}^{T} x_{i,t,s} = 1, \ \forall i \in I_{\bar{t}} \tag{5.18}$$

$$\sum_{s} \sum_{t=\bar{t}}^{T} (t * x_{i,t,s}) \geq a_i + r, \ \forall i \in I_{\bar{t}} \tag{5.19}$$

$$\sum_{s} \sum_{t=\bar{t}}^{T} ((t + l_s) * x_{i,t,s}) \leq a_i + r + d_i, \ \forall i \in I_{\bar{t}} \tag{5.20}$$

$$\sum_{t=\bar{t}}^{T} x_{i,t,s} = k_{i,s} \ \forall i \in I_{\bar{t}}, \forall s \tag{5.21}$$

$$x_{i,t,s} \in \{0,1\}, y_{j,t,s} \in [0,1], z_{j,t,s} \in \{0,1\} \ , \ z_{0,t,s} = z_{m,t,s} = 0 \tag{5.22}$$

As before, the total cost of all consolidations are minimized in (5.13), but with the new cost function representation. (5.14) and (5.15) construct the linear combination of the total weight of each consolidation and (5.16) and (5.17) impose the SOS2 property for each consolidation. Similar to the earlier MILP, (5.18) ensures that each order is assigned to only one consolidation. (5.19) makes sure that the order is not sent before it is available. (5.20) takes care that the order arrives before its deadline. Again constraints (5.21) ensures that the customer's chosen service type is not violated. As the y_js are not binary variables, strictly speaking we now have a mixed integer linear program.

5.1 Implementation

The mixed integer linear program presented in (5.13)-(5.22) is partly a 0-1 integer program, which is in general well known to be NP-hard (Karp, 1972). Hence, at first glance it does not seem a good idea to solve the program in an exact way by using the CPLEX library for C++. However, trying it out with CPLEX, some test instances (existing of a 100 days time line) take a couple of minutes but in general the solutions are rather quickly obtainable, see the results section. During the implementation, it occurred a few times that the CPLEX program declared a day instance as infeasible, while it is obvious that this instance is feasible. Unfortunately this issue could not be solved, it most likely has to do with the accuracy settings of CPLEX. Namely, when you take the day instance apart from the 100 days-instance and run it with a lower accuracy, there is no problem and the instance is solved. In order to be able to solve the test instances without the program to break down because of infeasibility, on these days it just checks for which orders deadlines are binding and sends only those orders, such that the program can proceed to the next day.

6 Test instances

There is no good test data available of AH-F because of privacy and incompleteness of the past dataset. Hence, general instances need to be generated in order to test the policies and MILP. So take in mind that the following assumptions do not always perfectly resemble reality for AH-F.

Origin-destination pair As the routing decisions are assumed to be given, the consolidation is done for an unique origin-destination pair (airport-to-airport). Hence the instances created will only consider orders for one (not mentioned) origin-destination pair, as the policies and MILP are applied to every origin-destination pair separately.

New order arrivals A common practice in the literature is to use a Poisson distribution to model the arrival of orders. Higginson and Bookbinder (1994) already mention this and (Bookbinder and Higginson, 2002), Çetinkaya and Bookbinder (2003) and Mutlu et al. (2010) build further on this assumption. Hence, the number of new orders that come in on a day in the test instances will be simulated using a Poisson distribution with several values for λ. Moreover, looking at the available data, weekly and monthly patterns reveal, so this is modeled as well by interchanging periods of high and low demand.

Time between arrival and ready for shipping It will be assumed that a fixed amount of 3 days is needed from the day that the order arrived until it is ready to be shipped. This is assumable as AH-F needs time to make their products ready or produce certain parts.

Deadlines It was mentioned by AH-F that their orders either have an urgent deadline of 1 day, or they have a more relaxed deadline, between 4 and 10 days. As no more information was given or can be retrieved from the past data, the simplified assumption will be made that the deadline equals 1, 4,..., 10 days with equal chance $\frac{1}{8}$. The deadline starts after an order is ready to be shipped.

Lead times The AOG service type and the routine service type both guarantee a different lead time. That is, the AOG service can ship faster from an origin to a destination than the routine service. How much faster depends on the origin-destination pair. For simplicity it will be assumed that the AOG service lead time is 1 day and the routine service lead time is 3 days.

Weights Similar to the common use of the Poisson distribution for the arrivals, the Gamma is a distribution that is often used to model the order weights Higginson and Bookbinder (1994), (Bookbinder and Higginson, 2002), (Çetinkaya and Bookbinder, 2003) , and (Mutlu et al., 2010). Especially because it is skewed to the left, so more probability is assigned to smaller weights, it is an interesting weight distribution to model consolidation (as combining several small orders will lead to cost savings). The past data shows that weights occur from very close to 0 kg to a couple of thousands kg, so several different α and β values for the Gamma distribution will be implemented that reflect such a weight diversity. However, all the distributions will have a mean of 240 kg, which reflects the real mean for a certain origin-destination pair in the past data.

Service types Each order has a predetermined service type. When the deadline is 1 day, inevitably the service type chosen is AOG. For the 4,..., 10 days deadlines, the AOG and routine service are chosen respectively chance $\frac{3}{7}$ and $\frac{4}{7}$, such that the overall AOG and routine service distribution is 50-50.

Type	New order arrivals	Weights
-1-1	Poisson(1)	Gamma(0.6, 400)
-1-2	Poisson(1)	Gamma(0.4, 600)
-1-3	Poisson(1)	Gamma(1, 240)
-2-1	Poisson(5)	Gamma(0.6, 400)
-2-2	Poisson(5)	Gamma(0.4, 600)
-2-3	Poisson(5)	Gamma(1, 240)
-3-1	5 days Poisson(3), 2 days nothing	Gamma(0.6, 400)
-3-2	5 days Poisson(3), 2 days nothing	Gamma(0.4, 600)
-3-3	5 days Poisson(3), 2 days nothing	Gamma(1, 240)
-4-1	14 days Poisson(1), 1 day Poisson(20)	Gamma(0.6, 400)
-4-2	14 days Poisson(1), 1 day Poisson(20)	Gamma(0.4, 600)
-4-3	14 days Poisson(1), 1 day Poisson(20)	Gamma(1, 240)

Table 6.1: *Properties of the different instances*

For each different combination of the assumptions in table 6.1, 10 test instances are created for a time line of 100 days (so in total 120 instances). In this way, the performance of the policies and the MILP can be tested over a longer time line. However, take in mind that the MILP can only use the information available up to the day that the planning is made, so the future information only becomes available for the planning when they really have come in. For the policies it is not of importance, as they do not look into the future anyways (only the decision for shipping today or not shipping today is made on every day).

Transportation costs The transportation cost functions are constructed in such a way that they represent a cost/kg rate table provided by DHL LLP for a all kinds of origin-destination pairs. The cost of the AOG service compared to the routine service in this provided table fluctuates from 120% up to more than 200%. However, when the AOG service is so expensive, that even for super heavy shipments the cost/kg rate is higher than the cost/kg rate for sending very small shipments with the routine service, shipping with AOG service can never generate cost savings. Hence for testing the instances, we will only compare an AOG service that is 120% more expensive than the routine service and an AOG service that is 140% more expensive. In this way, the cost/kg rate of the AOG service can still be better than the routine service for certain accumulated weights (although for the 140%, this margin is very narrow, and only appears for the heaviest weight cost/kg rate). See table 6.2 for the transportation cost functions in the breakpoint representation.

Weight b_j (kg)	Routine $f(b_j)$ (EUR)	120%-AOG $f(b_j)$ (EUR)	140%-AOG $f(b_j)$ (EUR)
0	0	0	0
0.00001	95	114	133
50	95	114	133
90	170	204	238
100	170	204	238
265	450	540	630
300	450	540	630
490	735	882	1029
500	735	882	1029
985	1450	1740	2030
1000	1450	1740	2030
2800	4050	4860	5670
3000	4050	4860	5670
100000	135000	162000	189000

Table 6.2: *Breakpoints for the Routine, 120%-AOG & 140%-AOG service*

Type	Tot. weight (kg)	Tot.Orders	A (%)	B (%)	Imm./Custom./MILP(%)
-1-1	24002	102	12.0	34.2	64.8
-1-2	22411	98	11.6	34.8	61.1
-1-3	23397	103	12.3	34.0	63.0
-2-1	117703	502	12.3	74.0	62.7
-2-2	119729	494	12.2	73.4	63.1
-2-3	117517	495	13.2	75.2	62.3
-3-1	52878	219	12.3	57.6	62.7
-3-2	51387	214	11.1	51.7	61.5
-3-3	53621	224	13.1	59.9	61.2
-4-1	51357	213	13.6	75.0	64.6
-4-2	50408	214	12.1	70.1	62.0
-4-3	50650	210	12.8	70.8	61.6

Table 6.3: *100 days-instances characteristics, averaged over the 10 instances with the same properties. The last 3 columns specify how many of the total orders are, on average, sent with the AOG service for the different policies/MILP. Obviously, the AOG policy sends always 100% of the orders with AOG.*

7 Results

By the way the instances have been generated, it can clearly be seen that the percentages sent by the AOG service of routine time policy A are due to the probability $\frac{1}{8}$ of having an order with a deadline of one day. Also it is remarkable that routine time policy B sends for certain instances more orders by the AOG service then the immediate shipping policy, but (as can be seen in the next tables) still generates cost savings.

Table 7.1 summarizes the total costs made per policy/by the MILP on the test instances for the transportation cost functions with the AOG 120% and the AOG 140% more expensive than the routine service.

Clearly, for AOG 120%, the routine time policy A performs best (as a matter of fact, not only on average, but also on every instance individually). The AOG time policy performs for some type of instances even worse than the immediate shipping policy. The routine time policy B and the customer chooses policy realize cost savings on average, but do not perform better than the routine time policy A. The MILP outperforms the customer chooses policy on average, so it would be a better consolidation solution in case that the service type chosen can not be neglected. As a matter of fact, the MILP generates more cost savings than the customer chooses policy for 117 of the 120 individual instances.

For the AOG 140% table, the routine time policy A jumps out as the best policy for cost savings (again as well for every instance individually). The AOG time policy performs always worse than immediate shipping and the routine time policy B and the customer chooses policy realize cost savings, but do not perform better than the routine time policy A. Once again, the MILP realizes more cost savings than the customer chooses policy on average. For the same three individual instances as before it performs slightly worse.

The policies were implemented in Xcode 6.1.1 using C++ on a Macbook Pro with a 2 GHz Intel Core i7 processor and 4 GB 1333 MHz DDR3 RAM memory. Applying all the policies for the two different cost structures on all 120 test instances takes less than 3 seconds and applying all policies with one cost structure to just one 100 days-instance never takes longer than 0.03 seconds. Hence, applying the best consolidation policy should be no problem in practice for AH-F running time wise, even when different origin-destination pairs are at hand and the consolidation has to be applied separately to them.

The MILP was implemented in Visual Studio Express 2012 using C++ and the CPLEX library on a HP Compaq PC with a 3 GHz Intel(R) Core(TM)2 Duo CPU E8400 Processor and 4 GB RAM memory. Running times of the 100 days-instances fluctuate between 28 and 164 seconds. So, especially when daily instances get bigger (more orders per day) and when many origin-destination pairs need to be solved, running time could become quite large compared to the policies running times.

Type	AOG (%)	A (%)	B (%)	Customer (%)	MILP (%)	Imm. (EUR)
-1-1	97.5	85.0	87.4	93.9	92.3	42693
-1-2	96.7	84.7	86.8	92.8	90.4	40042
-1-3	99.3	87.0	90.0	94.9	93.1	41070
-2-1	100.0	86.7	96.9	96.2	92.4	196698
-2-2	100.0	87.1	96.4	96.1	91.8	199469
-2-3	100.6	87.2	97.6	96.8	92.9	195765
-3-1	100.4	87.1	94.1	96.1	94.1	89542
-3-2	99.5	87.0	93.2	95.4	92.3	87516
-3-3	100.9	87.7	95.6	96.7	95.1	90391
-4-1	100.1	87.1	96.5	96.6	95.3	85503
-4-2	100.1	86.8	95.8	96.2	94.6	84231
-4-3	101.3	87.8	96.8	96.8	95.9	83327

Type	AOG (%)	A (%)	B (%)	Customer (%)	MILP (%)	Imm. (EUR)
-1-1	102.0	77.9	83.1	93.9	92.4	47644
-1-2	102.0	78.3	82.7	92.7	90.5	44305
-1-3	104.3	80.7	86.8	94.9	93.2	45630
-2-1	105.0	80.2	98.5	96.2	92.5	218597
-2-2	105.0	80.6	97.9	96.2	92.1	221602
-2-3	105.6	80.5	99.3	96.8	93.0	217625
-3-1	105.7	80.6	93.6	96.1	94.1	99242
-3-2	104.7	80.6	92.5	95.4	92.5	97057
-3-3	106.2	81.3	95.7	96.8	95.2	100178
-4-1	104.8	80.1	97.7	96.6	95.3	95341
-4-2	104.9	79.9	96.7	96.2	94.7	93822
-4-3	106.6	81.3	98.2	96.8	96.0	92407

Table 7.1: *100 days-instances performances on the policies with 120% and 140% AOG cost structure, averaged over the 10 instances with the same properties. The columns compare the average total costs of the policies to the immediate shipping policy, for which the hard value in EUR is given in the last column.*

8 Discussion

Of all policies, the routine time policy A outperforms all other policies by far, at least for the test instances used. The cost structure ratio between the routine and AOG services used are already very optimistic, as in the real data also 200% difference between the cost/kg rates exists. For such cost structures, the routine time policy A would even be more favored over the other policies that involve more AOG shipments. If the arrivals of orders or the weight distribution would be different, it could happen that a policy with more AOG shipments would outperform the routine policy A. At least for AH-F, often even less orders arrive over time for most origin-destination pairs, so then routine time policy A will most probably still work best. So as an advice for AH-F, the implementation of routine time policy A could lead to a lot of cost savings, if it would be possible to neglect the service type chosen by the customer. Hence it would be good to investigate if the customer always needs the AOG service type and if new agreements with their customers can be made about how their orders are shipped.

In case that the chosen service type really can not be neglected, the customer chooses policy and the MILP solution are a better alternative, although it will be more time consuming to calculate, especially when different origin-destinations pairs need to be calculated. Moreover, considering that in practice orders are coming in during the whole day, we would actually need to optimize the system more often than once a day with the rolling horizon method. Then the calculations might take too long in practice. The fact that in practice the system has to be reconsidered more often than once a day is not so much an issue for the policies, as there are no difficult calculations at hand there. So when the service type has to be met strictly and the running times of the MILP become too long in practice, then the customer chooses policy would be a better alternative.

Notice that both the policies and MILP do not take into account that future orders, which are not known yet, could be forecastable. Especially if the MILP would be able to take expected orders arriving in the future into account, the solution would be different, as it would take into account that currently available orders could also be sent with the expected future orders. Designing a heuristic that can solve such a stochastic program would be something to consider in next research. Also taking inventory cost into account would have an effect on the solution of the MILP, because waiting will become less favored when it costs money to store orders. This as well would be good to consider in future research.

References

Attanasio, A., Fuduli, A., Ghiani, G., and Triki, C. (2007). Integrated shipment dispatching and packing problems: a case study. *Journal of Mathematical Modelling and Algorithms*, 6(1):77–85.

Bisschop, J. (2006). *AIMMS optimization modeling*. Lulu. com.

Bookbinder, J. H. and Higginson, J. K. (2002). Probabilistic modeling of freight consolidation by private carriage. *Transportation Research Part E: Logistics and Transportation Review*, 38(5):305–318.

Çetinkaya, S. and Bookbinder, J. H. (2003). Stochastic models for the dispatch of consolidated shipments. *Transportation Research Part B: Methodological*, 37(8):747–768.

Crainic, T. G. and Laporte, G. (1997). Planning models for freight transportation. *European journal of operational research*, 97(3):409–438.

DAmbrosio, C. (2010). Application-oriented mixed integer non-linear programming. *4OR*, 8(3):319–322.

Higginson, J. K. and Bookbinder, J. H. (1994). Policy recommendations for a shipment-consolidation program. *Journal of Business Logistics*, 15(1):87.

Karp, R. M. (1972). *Reducibility among combinatorial problems*. Springer.

Mutlu, F., Çetinkaya, S. i. l., and Bookbinder, J. H. (2010). An analytical model for computing the optimal time-and-quantity-based policy for consolidated shipments. *Iie Transactions*, 42(5):367–377.

Qin, H., Zhang, Z., Qi, Z., and Lim, A. (2014). The freight consolidation and containerization problem. *European Journal of Operational Research*, 234(1):37–48.

Song, H., Hsu, V. N., and Cheung, R. K. (2008). Distribution coordination between suppliers and customers with a consolidation center. *Operations Research*, 56(5):1264–1277.

Tyan, J. C., Wang, F.-K., and Du, T. C. (2003). An evaluation of freight consolidation policies in global third party logistics. *Omega*, 31(1):55–62.