Summary

Most election methods are subject to manipulation. A group of voters can manipulate an election if by misrepresenting their honest preferences they can secure an outcome they all prefer. We show that when a candidate would win a 1-man-1-vote electionwith an absolute majority, it is not possible to individually manipulate this election in favour of another candidate.This is because changing only one vote will never change the outcome of the election; hence no individual manipulation will be successful. On the contrary, if a candidate with an absolute majority would win a 1-best-1-worst election, depending on the preferences of the participating voters,it is possible to manipulate. This is proven by giving an example of a manipulableprofile with a winner with the absolute majority.

1.1 Introduction

It is well-known that elections are subject to manipulation. Almost every participant of a democratic election is aware of the fact that voting according to his honest preference is not always the best option. Take an election in which you like a certain candidate the best, but that candidate has no chance of winning. Voting for him would be the "sincere" strategy,but it may feel like throwing away your vote. In that situation, you might choose to vote for another less-favoured candidate that has more chance of winning, this is voting according to a "sophisticated" strategy.[1]

This paper analyses the manipulability of two different election methods. The two election methods, also referred to as voting schemes, discussed in this paper will be the 1-man-1-vote and the 1-best-1-worst election method. The first election is similar to the election of the House of Representatives; every voter can give one vote to one candidate. The alternative that gets the most votes, wins the election (plurality[2]). This is a straightforward and simple system to select a winner. The downside is that every voter can only indicate one candidate as its best alternative, thus just a small part of his preference over all alternatives is revealed.

Another election method is the 1-best-1-worst election. Although this election method is also quite straightforward, it reveals a little more of

---

1 The terms "sincere" and "sophisticated" strategy are introduced in Farquharson R. "Theory of voting" *Yale University Press, New Haven 1969* p. foreword,17

2 García-Lapresta J.L., Marley A.A.J, Martínez-Panero M. (2010) "Characterizing best-worst voting systems in the scoring context" Social Choice and Welfare p. 489

the preferences of the voters. Here, next to the positive vote for one candidate, every voter can also indicate their worst alternative, and give that candidate a negative vote (anti-plurality).

Both these election methods are manipulable, but the matter is in how manipulable they are. In a 1-best-1-worst election you can manipulate in two ways, by changing the positive vote and by changing the negative vote. While on the other hand in the 1-man-1-vote election, you can only change one vote and thus you also only have one possible way of manipulating. Because in the 1-best-1-worst election a voter has more ways of manipulating, our intuition is that this election is also more manipulable. A conclusive proof of this hypothesis has not been found, but the proofsgiven in this paper do strengthen our intuition that this is indeed the case.

The paper starts by sketching a picture of existing literature that has been written about voting theory, focusing on best-worst elections and on manipulability. The third section goes into deeper detail about manipulation of election methods. We distinguish between individual and group manipulation and show what we consider that manipulation of results is. The last section is about our own findings on manipulability. It describes the aim of our analysis and what results we found.

1.2 Literature study

García, Marley and Martínez (2009) characterized the best-worst voting systems. They advocate that because voters in past elections have been able to indicate their best choice, they must also be able to indicate their worst choice. That is why it should not give a problem to indicate one best and one worst choice in the 1-best-1-worst election. They say that there is some evidence that proves that voters can make their minds up relatively easy about their p-best and q-worst alternatives, for a "small" p and q, in comparison to order all different candidates. Ranking all different candidates could induce "a kind of psychological paralysis in the voter".[3]But on the other hand, if a voter only has weak preferences over the candidates, it may be better not to ask the voter to only select its best and worst choice because it could be that several candidates are tied.

A 1-best-1-worst [α, β] election, corresponds to an extended scoring rule of (α, 0, …, 0, −β). This means that the candidate that is the best choice gets a score of +α and the candidate that is the worst choice gets a score of −β. In this paper, the basic case is considered where α=β=1. This means that being some voter's best choice and another voter's worst choice will offset the positive and negative points for that candidate. García, Marley and Martínez advocate that every scoring rule gets characterized by anonymity and neutrality, meaning that there is equality under the voters and a "symmetric status for each alternative".[4]

Gibbard (1973) stated in his paper that a voting scheme must always select one single winner in every election, whichmeans that a voting scheme cannot allow for ties. This is an important property when considering manipulability, because a player can only make up his mind about his voting strategyif he can deduce exactly who would be the winner according to the preferences among the players. In a situation

---

[3] García-Lapresta J.L., Marley A.A.J, Martínez-Panero M. (2010) "Characterizing best-worst voting systems in the scoring context" Social Choice and Welfare p.488

[4] García-Lapresta J.L., Marley A.A.J, Martínez-Panero M. (2010)"Characterizing best-worst voting systems in the scoring context" Social Choice and Welfare 34:487-496

where a voting scheme allows for ties, the system would leave the outcome of the election unclear in the situation where multiple candidates are tied. Hence, it would also leave it unclear whether or not to manipulate.

At last we would like to refer to one of the most important and famous theorems in the voting theory. The Gibbard-Satterthwaite Theoremstates that every voting scheme with at least three outcomes is either dictatorial or manipulable.[5]A voting scheme is dictatorial if the outcome of the election depends solely on the preference of one candidate, the dictator. The name of this theorem refers to the two men that independently from each other found the same theorem.

1.3 Manipulation

This section goes into depth on the definition of manipulation and when a voter can manipulate. It first explains how the preference orderings of all voters are structured and what a profile is. Then it describes individual and group manipulability of a profile. At last it explains what we consider to be manipulability of a result, since one interesting property about this manipulation is described in Section 4.

Keep in mind that manipulating an election method in reality is much different than in this theoretical analysis in the last section. In reality it would require a lot of effort to keep track of the honest preference of all other voters and in cases where there are a high number of voters itis close to impossible to know what the outcome of an election would be. All we know about a manipulable election or profile is that it could be

---

[5]Gibbard A. (1973) “Manipulation of Voting Schemes: A General Result” Econometrica, Vol. 41, P.587-601

possible, if a voter would be willing and would know all the preferences of the other voters, to manipulate the election.

1.3.1 Preference orderings

Assumed is that every voter has its individual preference rank-order of the candidates. We use these preference orderings to establish if a voter will vote according to its sincere or sophisticated strategy.

A preference ordering will consists of an enumeration of the candidates in a specificorder.If a voter's preference for example isA B C, this means that he prefers candidate A over B and candidate B over C. So, a preference order will dictateexactly how a voter likes a certain candidate over the other candidates.

In the analysis in this paper we make two assumptions about every voter's preference.

1. *Transitivity* - We assume transitivity between the preferences. If candidate A is preferred to candidate B and B to C, then A is also preferred to C.

2. *No indifference* - We assume that a voter is not indifferent between two alternatives; he always likes the one over the other, or the other way around.

1.3.2 Individual manipulation

An individual can manipulate an election method, if he can secure an outcome he prefers by misrepresenting its true preferences and voting according to that sophisticated preference.

A voting scheme is a (mathematical) rule that determines a winner out of the set of candidates based solely on the individualpreferences of the voters among the alternative candidates.

Let an election have *n* voters and *m* candidates. Every voter has its individual preference ordering P_i of all different candidates. The preference orderings of all voters together is called a profile P_N, with $P_N = (P_1, P_2, \dots, P_n)$. A voting scheme V is a function that assigns to this profile one alternative candidate, the winner. For each profile P_N, $V(P_N)$ is the outcome of the voting scheme if all voters would honestly represent their preferences $P_1, \dots, P_n$.A voting scheme V is manipulable for a voter k, if there exists a profile $P_N' = <P'_1, \dots, P'_n>$, with $P'_i = P_i$ except for i=k, such that

$$V(P_N') \quad P_k V(P_N).$$

Meaning that voter K prefers the outcome of the voting scheme, given that all other voters vote the same, when he changes his vote into one that does not truthfully represent his preference. So, voter K prefers voting according to preference P'_k instead of preference P_k and will adopt a sophisticated strategy.

Here we give an example of an individually manipulable profile in a 1-best-1-worst election with four different candidates denoted as A, B, C and D and four voters.

$$\begin{pmatrix} A & B & D & C \\ D & B & C & A \\ D & C & B & A \\ B & C & D & A \end{pmatrix}$$

In this profile every line states a preference ordering of a different voter, which means that a first voter has the preference on the first line of the matrix, a second voter the preference on the second line, and so on. Within these preference orderings, the candidates are ordered starting with the top-ranked candidate and ending with the bottom-ranked candidate. The result of this profile under this election rule would be $(p_A, p_B, p_C, p_D) = (1–3, 1–0, 0–1, 2–0) = (–2, 1, –1, 2)$[6] with p_A, p_B, p_C and p_D being the amount of points earned for candidate A, B, C and D, respectively. From this result it is clear that D would win the honest election. The first voter would have preferred to have candidate A or B as a winner. If he misrepresents its preference and gives his positive voteto candidate B instead of A, he can change the outcome into $(p_A', p_B', p_C', p_D') = (–3, 2, 0, 1)$. After this manipulation, candidate B will be the winner. Since the first voter, the manipulator, likes this outcome better, is this profile manipulable.

As just shown, in the 1-best-1-worst election method it is possible to individually manipulate. In the 1-man-1-vote election, the possibility of individual manipulation is much smaller sinceonly one voter would change only one (positive) vote.[7] Changing the result then is only possible if the result was a tie (favouring the wrong candidate) or will result in a tie.

[6] The result of a 1-best-1-worst profile can be calculated by counting the amount of first positions that every candidate has and subtracting the amount of last positions. We do that because, as mentioned in Section 2, we consider the 1-best-1-worst election in the basic case. Meaning that being a voter's best alternative offsets being a voter's worst alternative.

[7] If the original honest result was a tie between two (or more) candidates that you didn't vote for and that a candidate that you like less than (one of the) other(s) won, than you can change the result by voting for that other candidate. It is also possible to change the result if first a preferred candidate missed only one point to be in a tie and voting for that candidate will make him win the election. Besides these two possibilities, you cannot individually manipulate in a 1-man-1-vote election method.

1.3.4 Group manipulation

A group of voters can also manipulate together. Group manipulation is possible if by misrepresenting their preferences a group of voters can change the outcome of an election together into an outcome that they all prefer.

Let me show that it is possible to manipulate a 1-man-1-vote election with a group. Consider an election with three candidates A, B and C and nine voters. Four of these nine voters rank the candidates in order A B C, three voters rank the candidates in order B A C and two voters have the preference C B A. There areno voters with one of the other possible preference orderings. Hence the profile looks like this:

$$\begin{pmatrix} A & B & C\,(4) \\ B & A & C\,(3) \\ C & B & A\,(2) \end{pmatrix}$$

If all voters would vote according to their truthful preferences, the result of this election would be $(p_A, p_B, p_C) = (4, 3, 2)$[8], and candidate A would win. The third group of voters with preference C B A is not happy with this result, since the outcome of the election is their least-favourite. By voting for B, they will change the result into $(p_A', p_B', p_C') = (4, 5, 0)$ and candidate B would win the election. This means that by misrepresenting their preference C B A for preference B C A, the third group of voters could successfully manipulate the result of the election into an outcome that they all preferred. Hence, the profile in this example is group manipulable under the 1-man-1-vote election.

To be clear; if a profile is individually manipulable, then it is by definition also group manipulable. But if a profile is group manipulable,

---

[8] The result of a 1-best-1-worst profile can be calculated by counting the amount of first positions that every candidate has.

it does not have to be individually manipulable. This means that group manipulability is a weaker form of manipulability and that you can more easily find a profile that is group manipulable.

1.3.5 Manipulation of results

Next to the individual and group manipulation, we will also discuss the manipulability of a result. We say that a result of an election method is manipulable if there is at least one profile that fits to that result, which is manipulable.

An example of a manipulable result for the 1-man-1-vote election is (p_A, p_B, p_C) = (4, 3, 2), like the example in section 3.3, and an example of a manipulable result for the 1-best-1-worst election is the example in section 3.1, (p_A, p_B, p_C, p_D) = (−2, 1, −1, 2). It is clear that these results are manipulable, because in the previous sections we already gave examples of a manipulable profile fitting to these results.

1.4 Findings

The aim of this paper is to compare the manipulability of the 1-man-1-vote election to the1-best-1-worst election. To prove our intuition that the 1-best-1-worst election is more manipulable than the 1-man-1-vote election, is not an easy task. A first problem is the immensely large number of different profiles that can exist in an election. For example an election with five candidates and six voters would already result in almost 30 billion ($5!^6$) different profiles. Not to mention an election with thousands or millions of voters and more than five candidates. This makes it difficult to generalize the manipulability among all possible profiles.

Also a qualitative analysis of these profiles based on manipulability is challenging. As explained in the previous section is the possibility of individual manipulability in the 1-man-1-vote election small. But there are some profiles that are individually manipulable in the 1-man-1-vote election and not in the 1-best-1-worst. That makes the proof of our hypothesis more complex, since that shows that manipulability in 1-man-1-vote election does not immediately implicate also manipulability in the 1-best-1-worst election.

An example of a profile like that is:

$$\begin{pmatrix} A & C & B & D \\ D & A & B & C \\ D & A & C & B \\ B & A & C & D \end{pmatrix}$$

Considering a tie-breaking rule that lets the candidate with the lowest letter in the alphabet win the election, this profile is manipulable in the 1-man-1-vote election for the fourth voter.[9]In the 1-best-1-worst election on the other hand, no voter can manipulate this election to secure a better result, because changing his negative vote will make an even less-favoured candidate then candidate A win.[10] So even though our intuition is that the 1-best-1-worst election is more manipulable, in some particular cases it is exactly the other way around.

---

9 When he votes for candidate A instead of candidate B, he turns the result of the election into ($p_{A'}$, $p_{B'}$, $p_{C'}$, $p_{D-'}$) = (2, 0, 0, 2) and makes candidate A the winner. The honest result of the 1-man-1-vote election is (p_A, p_B, p_C,p_D) = (1, 1, 0, 2), with candidate D as the winner.

10 The honest result of the 1-best-1-worst election is (p_A, p_B, p_C,p_D) = (1, 0, –1, 0), with candidate A as the winner.If for example the third voter would change his preference D A C B into D C B A, this would not make candidate D but candidate B the winner, who he dislikes even more than the original winner candidate A.

In an attempt to get a grasp on proving our intuition, we tried to make life easier by just looking at elections with three candidates and we allowed for group manipulation.

The remainder of this section analyses relationships between the winner of an election having an absolute majority and the manipulability of the election. The first two findings are on group manipulability and they strengthen our intuition about the manipulability of the election methods. The third finding is on the manipulability of a result.

Basis of every proof:

Call the three different alternatives that the voters can vote for candidate A, B and C. Every voter can have one of the six matching possible preference orderings. All n voters are divided in sub-groups according to their preferences, like this:

Group A1:	A	B	C	(n_{A1})
Group A2:	A	C	B	(n_{A2})
Group B1:	B	C	A	(n_{B1})
Group B2:	B	A	C	(n_{B2})
Group C1:	C	A	B	(n_{C1})
Group C2:	C	B	A	(n_{C2})

This means that group A1 consists of n_{A1} voters that all prefer candidate A over B and B over C, group A2 consists of n_{A2} voters that all prefer candidate A over C and C over B, and so on.

Let n_A, n_B, n_C be the total number of voters that prefer candidate A, B or C, respectively, over the other two candidates. So, $n_A = n_{A1} + n_{A2}$, $n_B = n_{B1} + n_{B2}$ and $n_C = n_{C1} + n_{C2}$.

A candidate has an absolute majority, if more than half of the voters participating in the election like that candidate the best. This means that the sum of the voters that favour another alternative must be smaller than the number of voters that favour that candidate the best. For example in the case that there are three candidates, candidate A has an absolute majority if $n_A > n_B + n_C$.

A candidate wins the election if it has more points than all others, for example candidate A wins an election if you have that $p_A > p_B$ and $p_A > p_C$. Note that along the proofs in this section, we assume that there are no ties, so $p_A \neq p_B, p_B \neq p_C$ and $p_A \neq p_C$.

Important is that in all three proofs we investigate the case where candidate A would be the winner of the honest election. In the other cases, where either candidate B or C would have been the winner, the exact same proof can be drawn.

1.4.1 Theorem I

A profile with a candidate that has an absolute majority is never group manipulable under the 1-man-1-vote election rule.

Proof
The result of the 1-man-1-vote election at the truthful profile would be $(p_A, p_B, p_C) = (n_{A1} + n_{A2}, n_{B1} + n_{B2}, n_{C1} + n_{C2})$.

Using that candidate A is the winner of the honest elections you have that $p_A > p_B$ and $p_A > p_C$. Obviously, in the 1-man-1-vote election the (honest) winner is the one that has an absolute majority. Hence, you know as well that $n_A > n_B + n_C$.

Under this election rule, the groups that would want to manipulate are those who do not think candidate A is the best alternative, so group B1, B2, C1 and C2, the four groups that have B or C as a first preference. Group A1 and A2 would always go for the honest strategy, since their favourite candidate would win.

Of the four groups that potentially want to manipulate, two groups cannot positively change the outcome for themselves. The honest winner, candidate A, is the second-favourite of group B2 and C1. They cannot make the results better for their favourite candidate, because changing their honest vote would only give this candidate less points. Since the honest result is the best alternative, they will not manipulate.

Group B1 and C2 on the other hand, do have the incentive to manipulate to make their second-favourite win. We will show that although they have the incentive to manipulate, they will never succeed to actually change the outcome.

If either group B1 or C2 tries to manipulate, they would change their vote for their most-preferred candidate into a vote for their second-favourite. A manipulation works if it changes the outcome of the election. This is when either $p_A' < p_B'$ after manipulation of B1 or $p_A' < p_C'$ after manipulation of C2. Let us sketch what would happen if group B1 and C2 would try to manipulate.

1) Group B1 manipulates to make candidate C the winner and changes its preference from B C A into C B A. This changes the received points into:

$(p_A', p_B', p_C') = (n_{A1} + n_{A2}, n_{B2}, n_{C1} + n_{C2} + n_{B1})$.[11]

Because of the absolute majority you know that:

$p_A' = n_{A1} + n_{A2} = n_A > n_B + n_C = n_{B1} + n_{B2} + n_{C1} + n_{C2} > n_{C1} + n_{C2} + n_{B1} = p_C'$

and

[11] Note that even if $P_{A,B,C}' = p_{A,B,C}$, we will use the notation $p_{A,B,C}'$ for the amount of points for that candidate after manipulation.

$p_A' = n_{A1} + n_{A2} = n_A > n_B + n_C = n_{B1} + n_{B2} + n_{C1} + n_{C2} > n_{B2} = p_B'$

So, $p_A' > p_C'$ and $p_A' > p_C'$, which makes the manipulation unsuccessful.

2) Group C2 manipulates to make candidate b the winner and changes its preference from C B A into B C A. This changes the received points like this:

$(p_A', p_B', p_C') = (n_{A1}+n_{A2}, n_{B1}+n_{B2}+n_{C2}, n_{C1})$.

Because of the absolute majority you know that:
$p_A' = n_{A1}+n_{A2} = n_A > n_B + n_C = n_{B1}+n_{B2}+n_{C1}+n_{C2} > n_{B1}+n_{B2}+n_{C2} = n_B'$
and
$p_A' = n_{A1}+n_{A2} = n_A > n_B + n_C = n_{B1}+n_{B2}+n_{C1}+n_{C2} > n_{C1} = n_B'$
So, $p_A' > p_C'$ and $p_A' > p_B'$.

The fact that candidate A has the absolute majority makes that both group B1 and C2 cannot change the result. Hence, if there is a candidate in the 1-man-1-vote election that has an absolute majority, then that election is not manipulable.

1.4.2 Theorem II

A profile with a candidate that has an absolute majority can be as well group manipulable and not group manipulable under the 1-best-1-worst election rule.

Proof

The outcome of the 1-man-1-vote election at the truthful profile would be $(p_A, p_B, p_C) = (n_{A1} + n_{A2} - n_{B1} - n_{C2}, n_{B1} + n_{B2} - n_{A2} - n_{C1}, n_{C1} + n_{C2} - n_{A1} - n_{B2})$.

Using that candidate A is the winner of the honest elections you have that $p_A>p_B$ and $p_A>p_C$. Because the winner has an absolute majority, you also know that $n_A>n_B + n_C$.

In the 1-best-1-worst situation, group B2 and C2 can manipulate to make candidate b the winner and group B1 and C1 can manipulate to make candidate c the winner. The election is called manipulable if at least one of these groups can successfully manipulate. For the manipulation to be successful, after manipulating either p_B' or p_C' should have become bigger than p_A'.

1) Group B2 and C2 can manipulate by giving candidate B the positive point and candidate A the negative point, so misrepresenting both their preferences into B C A. This would change the points assigned to the candidates into:
   $(p_A', p_B', p_C') =$
   $(n_{A1} + n_{A2} - n_{B1} - n_{C2} - n_{B2},\quad n_{B1} + n_{B2} + n_{C2} - n_{A2} - n_{C1},\quad n_{C1} - n_{A1})$.
   If we have that $n_{A1} + n_{A2} - n_{B1} - n_{C2} - n_{B2} < n_{B1} + n_{B2} + n_{C2} - n_{A2} - n_{C1}$ (i), these two groups can manipulate.

2) Group B1 and C1 can manipulate by giving candidate C the positive point and candidate A the negative point, so misrepresenting both their preferences into C B A. This would change the points assigned to the candidates into:
   $(p_A', p_B', p_C') =$
   $(n_{A1} + n_{A2} - n_{B1} - n_{C1} - n_{C2}, n_{B2} - n_{A2}, n_{B1} + n_{C1} + n_{C2} - n_{A1} - n_{B2})$.
   If we have that $n_{A1} + n_{A2} - n_{B1} - n_{C1} - n_{C2} < n_{B1} + n_{C1} + n_{C2} - n_{A1} - n_{B2}$ (ii),these two groups can manipulate.

Although the winner has an absolute majority, it is still possible that both (i) and (ii) do not hold. If they both do not hold, then that profile is not manipulable under the 1-best-1-worst rule. If at least one of them does hold, then that profile is manipulable under the 1-best-1-worst rule.

I give an example of both types of profiles.

A manipulable profile

Consider a profile with: $n_{A1} = 10$, $n_{A2} = 0$, $n_{B1} = 1$, $n_{B2} = 3$, $n_{C1} = 1$, and $n_{C2} = 4$. Candidate A has an absolute majority, because $10 > 1 + 3 + 1 + 4 = 9$. The result obtained by a 1-best-1-worst election is $(p_A, p_B, p_C) = (5, 3, -8)$ which makes candidate A the winner of the honest election. Group B2 and C2 can manipulate this result into: $(p_A, p_B, p_C) = (2, 7, -9)$ by changing their preference into B C A and make candidate B the winner. Hence, this is an example of a profile in which the winner had the absolute majority, which is manipulable. Note that in this profile inequality (i) holds.

A non-manipulable profile

Consider a profile with: $n_{A1}=n_{B1}=n_{C1}=0$. Because group B1 and C1 are empty, we only have to investigate the manipulation of group B2 and C2. We are interested in the relationship R between p_A' and p_B' after manipulating in favour of candidate B.

$$n_{A1}+n_{A2}-n_{B1}-n_{C2}-n_{B2} \quad R \quad n_{B1}+n_{B2}+n_{C2}-n_{A2}-n_{C1}$$
$$\Leftrightarrow \quad n_{A2} - n_{C2} - n_{B2} \quad R \quad n_{B2} + n_{C2} - n_{A2}$$
$$\Leftrightarrow \quad 2n_{A2} \quad R \quad 2(n_{B2} + n_{C2})$$

Because candidate a has an absolute majority of the votes, you know that:

$$n_A = n_{A2} > n_B + n_C = n_{B2} + n_{C2}$$
$$\Leftrightarrow \quad 2n_{A2} > 2(n_{B2} + n_{C2})$$

That last inequality implies that the relationship R must be ‘bigger’. So, p_A’ will always be bigger than p_B’, even after manipulating for B. This shows that also group B2 and C2 cannot manipulate in this profile. Hence, this profile is non-manipulable. Note that because that R is a ‘bigger’-relation, inequality (i) does not hold.

This proves that a profile with a winner of a 1-best-1-worst election method that has the absolute majority, can be both group manipulable and not group manipulable.

1.4.3 Theorem III

A result obtained by a 1-man-1-vote election, with a winner that does not have the absolute majority of the votes, is manipulable.

Proof

The result of the 1-man-1-vote election at the truthful profile would be $(p_A, p_B, p_C) = (n_{A1} + n_{A2}, n_{B1} + n_{B2}, n_{C1} + n_{C2})$.

Using that candidate A is the winner of the honest elections you know that $p_A>p_B$ and $p_A>p_C$. Because the winner does not have the absolute majority you know that $n_A<n_B + n_C$.

We prove this theorem by showing that there exists a characteristic profile fitting to every result that is always manipulable if the winner has no absolute majority.

This characteristic profile has the feature that $n_{B2}=0$ and $n_{C1}=0$. Here, all voters that favour candidate b are in group B1 and all voters that like candidate c the best are in group C2. This makes that all voters, except for the ones whose favourite alternative was candidate a, have the incentive to manipulate. They will manipulate by changing their vote into a vote for their second favourite. This manipulation works if it changes the outcome of the election. This is when either $p_A' < p_B'$ after manipulation 1 or $p_A' < p_C'$ after manipulation 2.

Let us sketch what would happen if group B1 and C2 would try to manipulate.

1) Group B1 manipulates to make candidate c the winner and changes its preference from B C A into C B A. This changes the received points into: $(p_A', p_B', p_C') = (n_{A1} + n_{A2}, 0, n_{C2} + n_{B1})$. Because of the absolute majority you know that: $p_A' = n_{A1} + n_{A2} = n_A < n_B + n_C = n_{B1} + n_{C2} = p_C'$ $p_C' = n_{B1} + n_{C2} \geq 0 = p_B'$
This makes C the new winner and the first manipulation successful.

3) Group C2 manipulates to make candidate b the winner and changes its preference from C B A into B C A. This changes the received points like this:

$(p_A', p_B', p_C') = (n_{A1} + n_{A2}, n_{B1} + n_{C2}, 0)$.

Because the winner has no absolute majority, you know that:

$p_A' = n_{A1} + n_{A2} = n_A < n_C + n_B = n_{B1} + n_{C2} = p_B'$

$p_B' = n_{B1} + n_{C2} \geq 0 = p_C'$

This makes B the new winner and the second manipulation successful.

Hence, for every result there is this characteristic profile that is manipulablein favour of candidate B and C, when the winner has no absolute majority. This makes that a result with a winner that has absolute majority is manipulable.

Note that the characteristic profile that we use here is not necessarily unique, i.e. there could be other manipulable profiles fitting to a result with the same properties.

1.5 Conclusion

Every election method has its ownparticular method to select a winner. While comparing election methods, there are a lot of things worth considering. In this paper the focus was on the manipulability of election methods. Our intuition was (and still is) that the 1-best-1-worst election is more manipulable than the 1-man-1-vote election. A conclusive proof of this hypothesis has not been found, but the results presented in the previous section do reinforce our intuition. We found a piece of the puzzle in the attempt of proving this hypothesis.

What we know now is that when you have a profile with a winner that has an absolute majority, that in the 1-man-1-vote election this profile definitely is not group manipulable, but in the 1-best-1-worst election, this profile can be as well group manipulable as not group manipulable. Important is to keep in mind that this analysis only covered elections with three candidates.

Interesting open problems are what would happen if you change the number of candidates, if you do not allow for group manipulability and to find more pieces of the puzzle to prove that the 1-best-1-worst election method is more manipulable than the 1-man-1-vote. At last it is also interesting to analyse the (group) manipulability of other election methods.

Winner with absolute majority	
1-man-1-vote	Profile not group manipulable
1-best-1-worst	Depending on profile, group manipulable and not group manipulable

References

Farquharson R. “Theory of voting” Yale University Press, New Haven 1969 p. foreword,17

García-Lapresta J.L., Marley A.A.J, Martínez-Panero M. (2010) “Characterizing best-worst voting systems in the scoring context” Social Choice and Welfare

Gibbard A. (1973) “Manipulation of Voting Schemes: A General Result” Econometrica, Vol. 41, P.587-601